Smoothed Particle Hydrodynamic Simulations of Viscous Accretion Discs Around Black Holes[†]

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ABSTRACT
Viscous Keplerian discs become sub-Keplerian close to a black hole since they pass through sonic points before entering into it. We study the time evolution of polytropic viscous accretion discs (both in one and two dimensional flows) using Smoothed Particle Hydrodynamics. We discover that for a large region of the parameter space, when the flow viscosity parameter is less than a critical value, standing shock waves are formed. If the viscosity is very high then the shock disappears. In the intermediate viscosity the disc oscillates very significantly in viscous time-scale. Our simulations indicate that these centrifugally supported high density region close to a black hole plays an active role in the flow dynamics, and consequently, the radiation dynamics.

Key words: accretion, accretion discs – black hole physics – shock-waves – hydrodynamics

1. INTRODUCTION

One of the convincing ways to understand the nature of accretion processes on black holes is to use numerical techniques. This is because the steady solutions of analytical models are too simplistic and they usually concentrate on steady state solutions. The works present in the literature, which study the possibility of sub-Keplerian accretion flows are mainly carried out for inviscid flows (Hawley, Smarr & Wilson, 1984; Molteni, Lanzafame & Chakrabarti, 1994; [hereafter MLC94] Nobuto & Hanawa, 1994; Molteni, Ryu & Chakrabarti, 1996 [hereafter MRC96]; Ryu, Chakrabarti & Molteni, 1997 [hereafter RCM97]) although some simulations of viscous *Keplerian* discs are also present (Eggum, Coroniti & Katz, 1988). In Chakrabarti & Molteni (1995, hereafter CM95; and references therein), the time evolution of *one dimensional isothermal* viscous transonic flows (VTFs) has been studied. It was particularly noted that in some region of the parameter space, when the viscosity is smaller, the flow developed centrifugal pressure supported shock waves as is expected from the steady solutions (Chakrabarti, 1990; hereafter C90). As viscosity is increased, the angular momentum transport rate in the post-shock region is enhanced compared to the rate in the pre-shock flow and as a result the shock propagates outwards, and the flow becomes sub-sonic. Further out, the flow can join with a Keplerian accretion disc.

In the present paper, we study the behaviour of the sub-Keplerian VTFs when the flow is neither isothermal nor restricted only to the equatorial plane as in CM95. We examine the behaviour of the more general polytropic thick, viscous accretion discs. We find that even in two dimensional thick discs, shocks form and the steady shock location increases with viscosity as in one dimensional study of CM95. We also observe, to our surprise, that beyond a critical viscosity, when the steady shock is not expected, the flow forms an unsteady shock which periodically evacuates the disc. Beyond another critical viscosity (keeping other parameters unchanged), where the flow passes only through the inner sonic point, the shock disappears and only the smooth sub-Keplerian disc (originating from a Keplerian disc) remains. The presence oscillating solutions are clear indications of transitions from one topology of solutions to another. Presence of both steady and oscillating solutions is significant in view of the fact that spectral observations of black hole candidates do show quasi-periodic oscillations in between various stationary states. Simply put, two critical viscosities in the above context come into being for the following reasons: Beyond a critical viscosity, the shock conditions cease to be fulfilled even when the flow has two saddle type sonic points. The flow passes through both, using unsteady shocks. Beyond yet another, and higher value of critical viscosity, the outer saddle type sonic point cease to exist altogether and flow smoothly passes through the inner sonic point. The general behaviour of inviscid and viscous accretion flows and how these critical viscosities can be identified, are discussed in C90 and Chakrabarti (1996, hereafter C96) respectively.

The organization of the present paper is the following: In the next Section, we briefly discuss the theory of viscous transonic discs. In §3, we present the simulation results in one dimensional viscous discs. In §4, we present the results in two dimensional viscous discs. In §5, we discuss astrophysical importance of our solutions and present concluding remarks.

2. MODEL EQUATIONS

We model the nonrotating central compact object using Paczyński & Wiita (1980) potential. The inclusion of viscosity in the form of Shakura-Sunyaev prescription (or, other viscosity prescriptions as in Chakrabarti & Molteni, 1995) implies that the specific angular momentum λ is not constant everywhere, rather it is efficiently transported outward at the rate determined by the magnitude of viscosity.

We measure all distances, velocities and timescales in units of $2GM/c^2$, c and $2GM/c^3$ respectively. Below, we provide the Lagrangean formulae for the two-dimensional fluid dynamics equations for SPH in cylindrical coordinates. The mass conservation equation is:

$$\frac{D\rho}{Dt} = -\rho\nabla\mathbf{v}$$

(here, $\frac{D}{Dt}$ is the comoving derivative)

The momentum equation is:

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho}\nabla P + \mathbf{g} + \frac{v_\phi^2}{r}\widehat{r} - \frac{v_\phi v_r}{r}\widehat{\phi} + \frac{1}{\rho}\nabla\overleftrightarrow{\tau}$$

Where $\widehat{r}$ is the radial direction vector, $\widehat{\phi}$ is the tangential vector and,

$$\mathbf{g} = -\frac{1}{2(R-1)^2}\frac{\mathbf{R}}{R},$$

$$g_r = -\frac{1}{2(R-1)^2}\frac{r}{R},$$

$$g_z = -\frac{1}{2(R-1)^2}\frac{z}{R},$$

here, $R = r\widehat{r} + z\widehat{z}$

$$R = \sqrt{r^2 + z^2}$$

The radial momentum equation becomes,

$$\frac{Dv_r}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + g_r + \frac{v_\phi^2}{r}.$$

The vertical momentum equation becomes,

$$\frac{Dv_z}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g_z,$$

The tangential momentum equation becomes,

$$\frac{Dv_\phi}{Dt} = -\frac{v_\phi v_r}{r} + \frac{1}{\rho}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\phi}\right)\right]$$

where,

$$\tau_{r\phi} = \mu r\frac{\partial\Omega}{\partial r},$$

$$\Omega = \frac{v_\phi}{r}.$$

For the Shakura Sunyaev turbulent viscosity we have used,

$$\mu = \alpha\cdot\rho\cdot c_{sound}\cdot Z_{disc}$$

where the vertical thickness Z_{disc} is estimated from the assumption of the vertical equilibrium condition:

$$Z_{disc} = \frac{2}{\gamma}\cdot c_{sound}\cdot r\cdot(r-1)^2.$$

One may use the conventional energy equation as,

$$\frac{D\epsilon}{Dt} = -\frac{P}{\rho}\nabla\mathbf{v} + \frac{\Phi}{\rho} = -\frac{P}{\rho}\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\mu}{\rho}\left[r\frac{\partial\Omega}{\partial r}\right]^2,$$

where, ϵ is the specific thermal energy, and Φ is the usual viscous dissipation term. However, it turns out that a better accuracy is achieved if the sum total of specific kinetic and thermal energies are used instead. This is because this quantity is exactly conserved (since it can be put in a symmetric form so that pair of particles exchange equal amounts of energy, e.g. Monaghan 1985). So we prefer to use the following version of the energy equation:

$$\frac{D}{Dt}\left(\epsilon + \frac{1}{2}\mathbf{v}^2\right) = -\frac{P}{\rho}\nabla\mathbf{v} + \mathbf{v}\cdot\left(\frac{D\mathbf{v}}{Dt}\right) + \frac{1}{\rho}\nabla\left(\overleftrightarrow{\tau}:\mathbf{v}\right)$$

with

$$\left(\frac{D\mathbf{v}}{Dt}\right) = -\frac{1}{\rho}\nabla P + \mathbf{g}$$

where $P = (\gamma - 1)\rho\epsilon$ is the equation of state of ideal gas, $\mathbf{g}$ is the gravitational acceleration. $\overleftrightarrow{\tau}:\mathbf{v}$ is the vector resulting from the contraction of the stress tensor with the velocity vector. We include only $\tau_{r\phi}$ (namely, the $r\phi$ component) since it is the dominant contributor to the viscous stress. We choose for simplicity the pseudo-Newtonian potential proposed by Paczyński & Wiita (1980), where $\Phi(r,\theta) = 1/2(r-1)$. A complete steady solution requires the equations of energy, angular momentum and mass conservation supplied by transonic conditions at the critical points and the Rankine-Hugoniot conditions at the shock.

Since a black hole accretion is necessarily supersonic at the horizon (Chakrabarti, 1990), and since the flows are sub-Keplerian at the sonic point even for generalized accretion models (C96), a flow must deviate from a Keplerian disc much before it enters into a hole. We verify this by performing numerous numerical simulations. If the viscosity is smaller than a critical value, and if the shock conditions are satisfied, then the shocks form, otherwise the shock becomes unstable and propagate outwards to make a subsonic disc which joins with a Keplerian disc. The reason for the shock to propagate outwards is that the rate of angular momentum transport in the pre and post-shock flows become completely different. In particular, the transport rate is higher in the subsonic post-shock region. Even when the shock is absent, the centrifugal barrier supported region close to the black hole behave very similar to the post-shock region, and hence the emission properties, of this shock-free disc also becomes similar (Chakrabarti, 1997).

In this context, the difference between the approach to obtain the steady solutions (as in C90, or in C96) and time dependent solutions is worth noting. While obtaining a steady solution, since the flow must pass through a sonic point, it is customary to choose it's location and the specific angular momentum of the flow at this point. The rest of the properties of the flow (such as the location where the flow deviates from a Keplerian disc, it's injection velocities

etc.) are determined as eigenvalues. In the time-dependent studies, on the other hand, we fix the injected parameters (such as the specific energy and angular momentum) while the location of the sonic point and the angular momentum at the sonic point are determined automatically as eigenvalues. That these two approaches produce the same result (whenever steady state solutions exist) has been verified repeatedly (See, first paragraph of the Introduction for reference.). In the present paper also, we shall fix the parameters of the injecting matter at the outer boundary.

The numerical simulations we perform are done using Smoothed Particle Hydrodynamics (SPH). The code has been tested several times against theoretical works (e.g., CM93, CM95; Sponholz & Molteni, 1994; MRC96; RCM97) and is found to be sufficiently accurate as far as the angular momentum conservation is concerned. In the next section we present results in one dimensions and in §4, we present results in 2 dimensional disc.

2. SIMULATION RESULTS IN ONE DIMENSIONS

Figure 1 shows stationary shock locations as a function of Shakura-Sunyaev viscosity parameter α in thin, rotating, accreting flows. The viscosity coefficient ν is related to the viscosity parameter α by (see, Shakura & Sunyaev, 1973) $\mu = \alpha \rho a Z_{disc}$. In upper left panel, we plot density $\rho(r)$, in upper right panel, we plot radial velocity $v_r(r)$, in lower left panel, we plot $\lambda(r)$ and the in lower right panel, we plot Mach number $M_x(r)$ [same as $M_r(r)$ in axial direction]. Matter with sound velocity $a = 0.12272$, injection velocity $v_{in} = 0.12319$ and specific angular momentum $\lambda = 1.8$ is injected at $X_{in} = 26.15$. Each result has been obtained starting with an inviscid flow $\alpha = 0.0$ and then increasing α up to $4.6 \cdot 10^{-4}$. In *each* case, standing shock solution was obtained. Note, that in this particular case, the shock disappears around $\alpha = 4.6 \times 10^{-4}$ which is the critical α here. The shock location shifts outwards as described in C90 and C96 with viscosity, and the angular momentum becomes more and more Keplerian. The results are indeed similar to Chakrabarti & Molteni (1995) solution of viscous isothermal flows. As viscosity is increased, the width of the shock also increases as it should be. For a different choice of injected parameters, the critical α could be very high as well (C96).

In Fig. 2, we use the injected angular momentum to be same as the marginally stable value $\lambda = 1.8363$ at the outer boundary located at $X_{in} = 50$. Injection velocity of matter and the sound speed are 0.157 and 0.0738 respectively. The stationary shock locations are seen in all the viscosity parameter values. The critical viscosity in this particular case is much higher, and therefore the shock continues to persist for the range of α used. The angular momentum distribution becomes closer to Keplerian much faster for higher values of viscosity parameters. However, the flow remains *sub-Keplerian* throughout. By suitably adjusting the injection parameters at the outer boundary one obtains the solutions reaching a Keplerian disc. This requires very large number of smoothed particles.

3. LOCATION OF STANDING SHOCKS IN THICK VISCOUS ACCRETION DISCS

In this Section, we describe the formation of standing shocks in rotating, *thick*, axisymmetric, accretion discs onto a black hole. We introduce viscosity to the inviscid solutions described in detail by Molteni, Lanzafame & Chakrabarti, 1994 (hereafter referred to as MLC94). In MLC94, the en- tire parameter space spanned by the specific energy ϵ and specific angular momentum λ was depicted which shows the boundary between shock and no-shock solutions in a flow in vertical equilibrium. It was observed that the theoretical predictions based on flow models in vertical equilibrium regarding the shock locations are generally in good agreement with the simulation results when appropriate effects of turbulence is taken care of.

As in the one-dimensional simulations presented in the previous section, we introduce viscosity gradually also in the two dimensional simulations, and in *each* case, we continue simulations till a stationary solution is reached. Secondly, unlike in MLC94, where the simulations were carried out with matter injected only in one quadrant, here we inject matter in two quadrants – both the upper and the lower side of the equatorial plane. As in MLC94, the accretion rate $\dot{M}$ being an eigenvalue of the problem, it is fixed by the choice of the input parameters (ϵ, λ). Therefore, matter density is chosen to be equal to unity. Since the accretion rate is an eigenvalue of the problem, it is automatically adjusted once the specific angular momentum and the specific energy are fixed at the outer boundary. This is true when a steady state is reached. When a steady state is not reached, such considerations do not apply.

Figure 3 shows the XZ location of the SPH particles of the inviscid accretion disc model. Particles with specific angular momentum $\lambda = 1.6$ and energy $\epsilon = 0.001955219$ (which corresponds to injected radial velocity and the sound speed as $v_{in} = 0.121122$ and $a_{in} = 0.058989$ at $X_{in} = 30$ respectively) are injected from the outer edge of the disc at $X = 30$. The results in the upper left panel is for inviscid flow. It is shown at $t = 2900$ when the flow has achieved a steady state. The total number of particles, whose size is $h = 0.3$, is 6146. A stationary shock is formed around $X \sim 5.2$ and weaker oblique shocks are also seen (e.g., MLC94). At the two wings of the hot post-shock flow (basically inside the centrifugal barrier and the funnel wall, see, MRC96) a subsonic cooler outflow is present, symmetrically both up and down which carry away of the order of 5% of the total mass of the disc. No turbulent motions are evident in this model in the narrow post-shock subsonic flow. Subsequent to the shock, the hot flow becomes supersonic again before entering inside the black hole.

In the other three panels we show the effect of the introduction of viscosity in the flow. Clearly, as in the one-dimensional case, higher viscosity causes higher differential angular momentum transport between the pre- and the post-shock solutions and as a result the shock is drifted away in the radial direction. The value of the viscosity parameter α is written on each panel. In the increasing order of viscosity, the shocks locations are $X_s = 6.0$, 7.5, and 10.5 respectively. Another point of interest: as viscosity is raised, the amount of outflowing matter in the wind is decreased. This is because of weakening of the centrifugal barrier of the ingoing matter. Lower viscosity causes matter to bounce from the barrier and fly away as winds. In higher viscosity, higher turbulences are also seen to form in the post-shock flow. This is because more matter from higher elevation falls on the equatorial plane and convert their potential energy to turbulent energy.

It is now widely believed (see, e.g., Chakrabarti, 1997 and references therein) that the observed X-rays in black

hole candidates are produced and reprocessed in the centrifugal barrier supported regions very close to the black holes. The soft photons released from the pre-shock Keplerian disc is reprocessed in the centrifugal pressure supported region (with or without shock) to become hard X-rays. Occasionally, radiations show quasi-periodic oscillations. Molteni, Sponholz & Chakrabarti (1996), and Ryu, Chakrabarti & Molteni (1997) suggest that these so-called QPOs observed in the compact objects could be due to the oscillations of the 'boundary layers', i.e., the centrifugally supported denser region. These oscillations form for those flow parameters where the theoretical analysis does not predict the formation of steady shocks, even though two sonic points exist. However, so far, no viscous flow was simulated. In Fig. 4a we show the first simulations of viscous flow shocks oscillate periodically. In the upper-left, upper-right and lower-left panels roughly half of the cycle is shown where the shock location decreases monotonically. In the lower-right panel, the shock drifted again outward. The parameters are same as in Fig. 3, but the viscosity is higher: $\alpha = 2 \times 10^{-3}$. The time period is around $T_{QPO} = 4000$ in units of $2GM/c^2$. Thus, in a stellar black hole of mass $10M_\odot$, the period would be around 0.4s, whereas in a supermassive black hole of mass $10^7 M_\odot$, the period would be, 10^6s or a few days. These time scales are comparable with the time scale of observed QPOs in black hole candidates. With a different disc input parameters (e.g., for different accretion rate or viscosity of the original disc) the periods will vary. Clearer evidence of the oscillation of the shock is shown in Fig. 4b, where the matter close to the equatorial place is collected and their Mach numbers are plotted. The behaviour of the shock on the equatorial plane is similar to what is seen in the one-dimensional simulations of Molteni, Sponholz & Chakrabarti, 1996.

Fig. 4b: Mach number of the particles close to the equatorial plane is plotted against radial distance at different phases of oscillation for the case shown in Fig. 4a

Figure 5 shows the variation of the number of simulation particles as a function of time in the case shown in Fig. 4a. Here the total number N and the number of sub-sonic particles N_{sub} (presumably, participating in the Compton reprocessing of the soft-photons from the Keplerian disc) are shown. The amplitude modulation is significant: 50 percent variation in total particle number, and more than 700 percent variation in the sub-sonic particle number. We believe that this is important: the observed significant $(10-100$ percent) variations in QPO cannot be explained away by any means other than such a dynamical variation of the X-ray emitting region.

4. CONCLUDING REMARKS

In the present paper, we have numerically studied the behaviour of viscous, transonic flows. There are several important conclusions: We show that the location of the centrifugally driven standing shock wave drifts away from the black hole and ultimately the shock disappears. This is valid both for one and two dimensional axisymmetric discs originating from Keplerian flows. This result generalizes the earlier study of Chakrabarti & Molteni (1995) for isothermal flows done in one spatial dimensions. Second most important conclusion is that the viscous flows also show shock oscillations in regions of the parameter space where the steady shock is not possible even when two saddle type sonic points are present. This generalizes earlier work of Ryu,

Chakrabarti & Molteni (1997) where non-viscous flow was studied in two dimensions, and that of Molteni, Sponholz & Chakrabarti (1996) where non-viscous flow was studied in presence of bremsstrahlung cooling. We believe that both these findings are very important in explaining the observed X-ray radiations from the black hole candidates. The drifting shock solutions present the insight of how a Keplerian disc is actually formed out of an original sub-Keplerian flow and how the centrifugal pressure supported region in the sub-Keplerian region around the black hole may act as the so-called Compton cloud. The dynamical oscillation of the centrifugal pressure supported shock wave (which, for the first time is shown here to be present even in a viscous transonic disc) produces right frequency and the amplitude of modulation to be taken a serious explanation of the quasi-periodic oscillations.

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Figure Captions

Fig. 1: Variation of density (upper-left), radial velocity (upper-right), angular momentum (lower-left) and Mach number (lower-right) with distance in a viscous transonic flow. Different curves are drawn for different viscosity parameters. As viscosity is increased, the shock is drifted outwards, until it disappears altogether beyond a critical viscosity parameter. The specific angular momentum with which matter is injected at the outer boundary is $\lambda = 1.8$

Fig. 2: Same as in Fig. 1 except that the specific angular momentum of the injected flow is chosen to be the same as the marginally stable value of 1.8363.

Fig. 3: Drifting of the steady shock location in a two dimensional, axisymmetric accretion flow as viscosity parameter is increased towards the critical value. See text for injected parameters.

Fig. 4a: Oscillation of the shocks with time when a steady shock is not predicted, yet two sonic points exist in a viscous transonic flow. These solutions provide viable explaination to the quasi-periodic modulation of the X-rays in blak hole candidates.

Fig. 5: Oscillation of the total amount of matter in the accretion disc with a shock wave. Total particle number N and subsonic particle number N_{sub} are plotted as functions of time. Parameters are same as those used to draw Fig. 4(a-b).

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